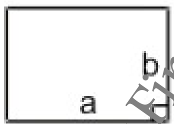
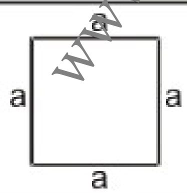
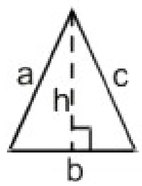
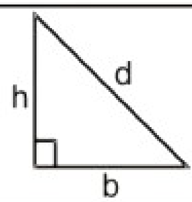
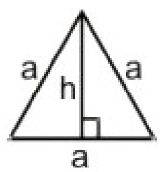
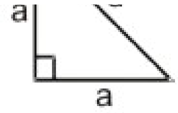
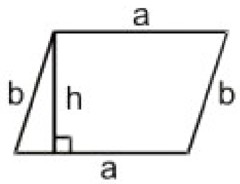
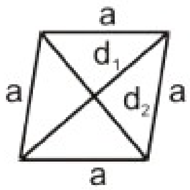
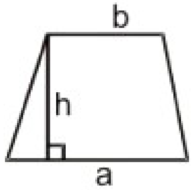
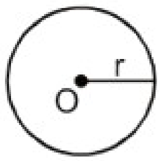
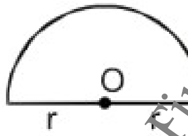
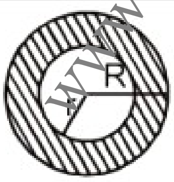
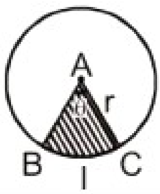
Areas and Mensuration

Areas and mensuration is a topic depends entirely on application of formulas. If you remember the formula, solving problems in this area is a cakewalk. For easy learning and to remember the formulas we provided them into a simple table. Before you move on to have a look at the solved examples, study the tables and try to understand the relevant formula

2 Dimensional Figures:

Two dimensional figures have only any two of length, breadth, and height. They have only areas but not volumes. Perimeter is a uni-dimensional measure. If you observe carefully, the power of the terms in the formulas of perimeter is 1, and of the terms in the areas is 2.

Name	Figure	Perimeter	Area
Rectangle		2 (a + b)	ab
Square		4a	a^2
Triangle		a + b + c = 2s	1 = $\frac{1}{2} \times b \times h$ 2 = $\sqrt{s(s-a)(s-b)(s-c)}$
Right triangle		b + h + d	$\frac{1}{2}$ bh
Equilateral triangle		3a	1. $\frac{1}{2}$ ah 2. $\frac{\sqrt{3}}{4} a^2$
Isosceles right			1

Isosceles right triangle		2a + d	$\frac{1}{2}a^2$
Parallelogram		2 (a + b)	ah
Rhombus		4a	$\frac{1}{2}d_1d_2$
Trapezium		Sum of its four sides	$\frac{1}{2}h(a + b)$
Circle		$2\pi r$	πr^2
Semicircle		$\pi r + 2r$	$\frac{1}{2}\pi r^2$
Ring (shaded region)		----	$\pi(R^2 - r^2)$
Sector of a circle		l + 2r where l = $(\theta/360) \times 2\pi r$	$\theta/360° \times \pi r^2$

Solved Examples

1. If sides of a triangle are 8 cm, 15 cm and 17 cm respectively. Find its area.

Area of the triangle when all the three sides are given = $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$

$s = \frac{8+15+17}{2}$ = 20 cm

Therefore, Area = $\sqrt{20 \times (20-8) \times (20-15) \times (20-17)}$

= $\sqrt{20 \times 12 \times 5 \times 3}$

= $\sqrt{4 \times 5 \times 3 \times 4 \times 5 \times 3}$ = 4 x 5 x 3 = 60 cm^2

Trick:

The triangle is right angle triangle as $17^2 = 15^2 + 8^2$

Therefore, Area of right angle triangle = $\frac{1}{2}$ x 8 x 15 = 60 cm^2

2. Two parallel sides of a trapezium are 4 cm and 5 cm respectively. The perpendicular distance between the parallel sides is 6 cm. Find the area of the trapezium.

Area of trapezium when height and two parallel sides are given = $\frac{1}{2} \times h \times (a+b)$ = $\frac{1}{2}$ x 6 x (4 + 5) = 27cm^2

3. If perimeter and area of a square are equal. Side of the square (in cm) is:

Given, $(\text{Side})^2$ = 4 x (Side)

Therefore, Side = 4 cm

4. The diameter of the wheel of a vehicle is 5 metre. It makes 7 revolutions per 9 seconds. What is speed of the vehicle in km/h?

Radius of the wheel = $\frac{5}{2}$ metre

Distance covered in 1 revolution = Circumference of the wheel = $2\pi r$ = 2 x $\frac{22}{7} \times \frac{5}{2}$ metre

Therefore, Distance covered in one second = 2 x $\frac{22}{7} \times \frac{5}{2} \times \frac{7}{9}$ metre

Therefore, Speed per hour = 2 x $\frac{22}{7} \times \frac{5}{2} \times \frac{7}{9} \times \frac{18}{5}$ = 44 km/h

5. The perimeter of a rhombus is 60 cm and one of its diagonal is 24 cm. Find the other diagonal of the rhombus.

Let ABCD is the rhombus whose diagonals BD and AC intersecting at point 'O'.

Side of rhombus = $\frac{1}{4}$ x Perimeter = $\frac{1}{4}$ x 60 = 15 cm.

Let BD = 24 cm

Then, BO = $\frac{1}{2}$BD = 12 cm

Now, AO = $\sqrt{AB^2 - BO^2} = \sqrt{15^2 - 12^2}$ = 9

Therefore, AC = 2 AO = 2 x 9 = 18 cm

6. A field is 40 metre long and 35 metre wide. The field is surrounded by a path of uniform width of 2.5 metre runs round it on the outside. Find the area of the path.

Remember the formula for the Area of path

= 2 x Width x [Length + Breadth + (2 x Width)]

= 2 x 25 x (40 + 35 + 2 x 2.5)

= 5 x (75 + 5) = 400 m^2

7. Find area of uniform path of width 2 metre running from centre of each side of the opposite side of a rectangle field measuring 17 metre by 12 metre.

Remember the formula for the Area of path

= Width of path x (Length of field + Breadth of field) - $(\text{Width of path})^2$

= 2 x (17 + 12) - $(2)^2$

= 58 - 4 = 54 m^2

8. A square and a rectangle each has perimeter 60 metre. Difference between the areas of the two figures is 16 square metre. Find length of the rectangle.

Side of the square = $\frac{60}{4}$ = 15 metre

Difference in areas of square and rectangle = 16m^2

Therefore, Increase and decrease in dimensions = $\sqrt{16}$ = 4 metre

Therefore, Sides of rectangle are (15 + 4) and (15 - 4) metre

Therefore, Length of rectangle = 19 metre

9. Find the radius of a circle whose area is equal to the sum of areas of three circles with radii 8 cm, 9 cm, 12 cm respectively.

Let radius of new circle is 'R' cm.

Then $\pi R^2 = \pi 8^2 + \pi 9^2 + \pi 12^2 = 64\pi + 81\pi + 144\pi = 289\pi$

Therefore, R^2 = 289 => R = 17 cm.

10. The are of the ring between two concentric circles, whose circumferences are 88 cm and 132 cm is:

Area of Ring = $\frac{1}{4\pi} \times (132^2 - 88^2) = \frac{1}{4\pi}$ x (132 + 88) x (132 - 88)

= $\frac{1}{4} \times \frac{7}{22}$ x 220 x 44 = 770 cm^2

11. Two poles whose heights are 11 metre and 5 metre stand upright in a field. If the distance between their feet is 8 metre, what is the distance between their tops?

Distance between feet of big pole and that of small pole = 8 metre

Difference between heights of two poles = 11 - 5 = 6 metre

Distance between the tops of poles = $\sqrt{6^2 + 8^2}$ = 10 metre

12. Find the are of the largest circle that can be inscribed in a square of 14 cm, a side.

Side of the square = Diameter of the circle = 14 cm

Therefore, Radius of the circle = 7 cm.

Area of the circle = $\pi r^2 = \frac{22}{7}$ x 7 x 7 = 154 cm^2.

13. A horse is tied to one of the corners of a square field whose side is 20 metre. If length of the rope is 14 metre, find the area of ungrazed field.

Area of square field = $(20\text{m})^2$ = 400 m^2

Area of the field grazed by the horse = $\frac{1}{4}\pi r^2$

= $\frac{1}{4} \times \frac{22}{7}$ x 14 x 14 = 154 m^2

Therefore, Field ungrazed = (400 - 154) = 246 m^2

14. A horses are tied one to each corner of a square with 14 m a side, and length of the rope is 7 m. Find the area of ungrazed field.

Ungrazed area = Area of square - Area of circle

= $14^2 - \frac{22}{7}$ x 7 x 7 = 196 - 154 = 42 m^2

Short-Cut Method:

Radius of circle = 7 m.

Therefore, Ungraged field = $\frac{6}{7} \times 7^2$ = 42 m^2

MCQ's

1. A rectangular carpet has an area of 60 sq.m. Its diagonal and longer side together equal 5 times the shorter side. The length of the carpet is :

a. 5 m

b. 12 m

c. 13 m

d. 14.5 m

Correct Option: B

Explanation:

Let the length = p metres and breadth = q metres

Then pq = 60

$\sqrt{p^2 + q^2} + p = 5q$

$\Rightarrow p^2 + q^2 = (5q - p)^2 \quad = 25q^2 + p^2 - 10pq$

As pq = 60,

$\Rightarrow 25q^2 - 10 \times 60 = 0$

$\Rightarrow q^2 = 25$ or q = 5

p = 60/5 = 12m

Length of the carpet = 12m

2. A rectangular carpet has an area of 120 sq.m and perimeter of 46 m. The length of its diagonal is :

a. 15 m

b. 16 m

c. 17 m

d. 20 m

Correct Option: C

Explanation:

Let length = a metres and breadth = b metres

Then, 2(a+b)=46 or (a+b) = 23

and ab = 120

Diagonal = $\sqrt{a^2+b^2} = \sqrt{(a+b)^2 - ab} = \sqrt{(23)^2 - 2\times120} = \sqrt{289} = 17m$

3. A parallelogram has sides 60 m and 40 m and one of its diagonals is 80 m long. Then, its area is :

a. 480 sq.m

b. 320 sq.m

c. $600\sqrt{15}$ sq.m

d. 450 $\sqrt{15}$ sq.m

Correct Option: C

Explanation:

AB = 60m, BC=40m or AC = 80m

s=$\frac{1}{2}(60+40+80)m = 90m$

(s-a) = 30m, (s-b) = 50m and (s-c)=10m

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

=$\sqrt{90\times30\times50\times10}\ m^2 = 300\sqrt{15}\,m^2$

Area of //gm ABCD = $600\sqrt{15}\,m^2$

4. If a square and a parallelogram stand on the same base, then the ratio of the area of the square and the parallelogram is :

a. greater than 1

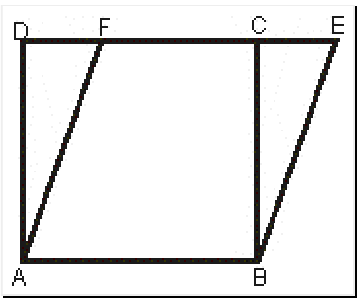
b. equal to 1

c. equal to $\frac{1}{2}$

d. equal to $\frac{1}{4}$

Correct Option: B

Explanation:



Let ABCD be the square and ABEF be the parallelogram.

Then, in right triangles ADF and BCE, we have AD = BC (sides of a square) and AF = BE (sides of parallelogram).

Therefore, DF = CE

$[DF^2 = AF^2 - AD^2 = BE^2 - BC^2 = CE^2]$

Thus, $\Delta ADF = \Delta BCE$

$\Rightarrow \Delta ADF + \square ABCF = \triangle BCE + \square ABCF$

$\Rightarrow$ Area of parallelogram ABEF = Area of ABCD

5. In a rhombus, whose area is 144 sq.cim, one of its diagonals is twice as long as the other. The length of its diagonals are :

a. 24 cm, 48 cm

b. 12 cm, 24 cm

c. $6\sqrt{2}$ cm, $12\sqrt{2}$ cm

d. 6 cm, 12 cm

Correct Option: B

Explanation:

$\frac{1}{2} \times x \times 2x = 144 \Rightarrow x^2 = 144$ or x = 12

Length of diagonals = 12cm, 24cm

6. The length of a rope by which a cow must be tethered in order that she may be able to graze an area of 9856 sq.m.is:

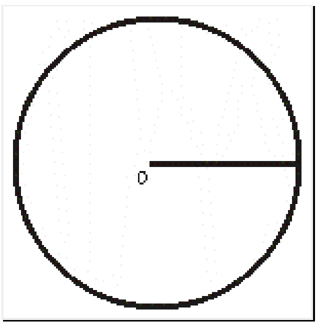
a. 56 m

b. 64 m

c. 88 m

d. 168 m

Correct Option: A



Explanation:

Grazing area is equal to the area of a circle with radius r.

$$\frac{22}{7} \times r^2 = 9856$$

Then $r^2 = (9856 \times \frac{7}{22})$

r = 56 m

7. The circumferences of two concentric circles are 176 m and 132 m respectively. What is the difference between their radii?

a. 5 metres

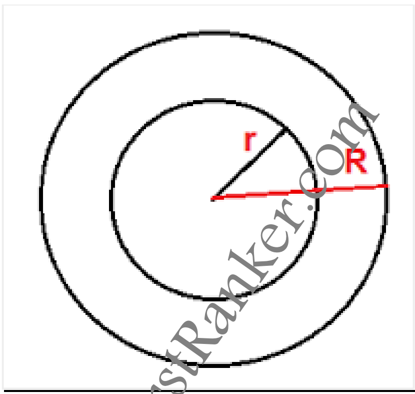
b. 7 metres

c. 8 metres

d. 44 metres

Correct Option: B

Explanation:



$$2\pi R - 2\pi r = (176 - 132)$$
$$\Rightarrow 2\pi(R - r) = 44$$
$$\Rightarrow (R - r) = \frac{44 \times 7}{2 \times 22} = 7\text{m}$$

8. The diameter of a circle is 105 cm less than the circumference. What is the diameter of the circle ?

a. 44 cm

b. 46 cm

c. 48 cm

d. 49 cm

Correct Option: D

Explanation:

$\pi d - d = 105 \Rightarrow (\pi - 1)d = 105$

$\Rightarrow (\frac{22}{7} - 1)$ d = 105

d = $(\frac{7}{15} \times 105)$ cm=49cm

9. A circle and a square have same area. The ratio of the side of the square and the radious of the circle is :

a. $\sqrt{\pi}$:1

b. 1 : $\sqrt{r}$

c. 1 : r

d. r : 1

Correct Option: B

Explanation:

$x^2 = \pi r^2 \Rightarrow \frac{x}{r} = \sqrt{\pi} = \sqrt{\pi} : 1$

10. The number of rounds that a wheel of diameter $\frac{7}{11}$ m will make in going 4 km, is :

a. 1000

b. 1500

c. 1700

d. 2000

Correct Option: D

Explanation:

Number of rounds = $\frac{4 \times 1000}{\frac{22}{7} \times \frac{7}{11}} = 2000$

11. A circular wire of radius 42 cm is cut and bent in the form of a rectangle whose sides are in the ratio of 6:5. The smaller side of the rectangle is :

a. 30 cm

b. 60 cm

c. 72 cm

d. 132 cm

Correct Option: B

Explanation:

Circumference = $(2 \times \frac{22}{7} \times 42)$ cm = 264 cm

Let the rectangle sides are 6x, 5x. Then circumference is

$2 \times (6x + 5x) = 264$ or x = 12

Smaller side of recatngle = 5x = 60 cm

12. If the diameter of a circle is increased by 100% , its area is increased by :

a. 100%

b. 200%

c. 300%

d. 400%

Correct Option: C

Explanation:

Original area = $\pi \times (\frac{d}{2})^2 = \frac{\pi d^2}{4}$

New area = $\pi \times (\frac{2d}{2})^2 = \pi d^2$

Increase in area = $(\pi d^2 - \frac{\pi d^2}{4}) = \frac{3\pi d^2}{4}$

Increase percent = $(\frac{3\pi d^2}{4} \times \frac{4}{\pi d^2} \times 100)\%$ = 300%

Shortcut:

You can use $(A + B + \frac{AB}{100})\%$ formula. Substitute A = B = 100

13.If the radius of a circle is reduced by 50%, its area is reduced by :

a. 25%

b. 50%

c. 75%

d. 100%

Correct Option: C

Explanation:

Original area = $\pi \times r^2$

New are = $\pi \times (\frac{r}{2})^2 = \frac{\pi r^2}{4}$

Reduction in area = $(\pi r^2 - \frac{\pi r^2}{4}) = \frac{3\pi r^2}{4}$

Reduction percent = $(\frac{3\pi r^2}{4} \times \frac{1}{\pi r^2} \times 100)\%$ =75%

Shortcut:

You can use $(A + B + \frac{AB}{100})\%$ formula. Substitute A = B = - 50.

14. If the circumference of a circle is 352 metres, then its area in m^2 is :

a. 9856

b. 8956

c. 6589

d. 5986

Correct Option: A

Explanation: